Roll No. | | | | | | | | | | | | | **Total No. of Pages : 02**

Total No. of Questions : 18

B.Tech. (IT) (2018 Batch) (Sem.–3)

MATHEMATICS-III

Subject Code : BTAM-304-18

M.Code : 76393

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

Write brief :

1. Express $\partial w/\partial r$ and $\partial w/\partial s$ in terms of r and s if

$$w = x + 2y + z^2, \quad x = \frac{r}{s}, \quad y = r^2 + \ln s, \quad z = 2r$$

2. Show that the function

$$f(x,y) = \begin{cases} \dfrac{2xy}{x^2 + y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

is not continuous at origin.

3. Find the local extreme values of the function $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$

4. Define convergence of a sequence and give an example of a convergent sequence.

5. State Leibniz's test for alternating series.

6. Determine for what values of a and b, the following differential equation is exact.

$$(y + x^3)dx + (ax + by^3)dy = 0$$

7. Find the integrating factor for the following differential equation

$$(5x^3 + 12x^2 + 6y^2)dx + 6xydy = 0$$

8. Give an example of a fourth order linear differential equation.

9. Find the solution of the differential equation $y''' - 4y' - 12y = 0$

10. If y_1 and y_2 are two linearly independent solutions of a second order linear differential equation, then what can you say about the general solution of this equation. Justify your answer.

SECTION-B

11. Find the volume of the region in the first octant bounded by the coordinate planes and the planes $x + z - 1$ and $y + 2z = 2$.

12. For what values of x does the following power series converge ?

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{2n-1}$$

13. Obtain the general solution and singular solution of the non-linear equation

$$y = xy' - (y')^3.$$

14. Solve the differential equation $y'' + 16y = 32 \sec 2x$ by using method of variation of parameters.

15. If $\sum_{n=1}^{\infty} |a_n|$ converges, then show that $\sum_{n=1}^{\infty} a_n$ also converges. Is the converse also true? Justify your answer.

SECTION-C

16. Find the extreme values of the function $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraints $x^2 + y^2 - 1 = 0$ and $x + y + z = 1$.

17. Test the convergence of the series (i) $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n - \ln n}$ (ii) $\sum_{n=1}^{\infty} \frac{8 \tan^{-1} n}{1 + n^2}$

18. Find the general solution of the equation $y''' - 4y' + 13y = 18e^{2x} \sin 3x$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.